Nonlinear electron-acoustic excitations in the presence of superthermal background electrons

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1. Introduction

Superthermal (accelerated) particles are often present in laboratory experiments [1], in the solar atmosphere [2] and in space plasma [3] environments. These plasmas are diagnosed by a high energy tail in their particle distribution, which is efficiently modelled by a generalized Lorentzian or κ-type distribution function. The parameter κ determines the high energy power law index (it approaches the Maxwellian distribution for $\kappa \to \infty$; see [4]). The κ-distribution function was argued to provide a *much better fit* to existing observations than a Maxwellian function [5, 6].

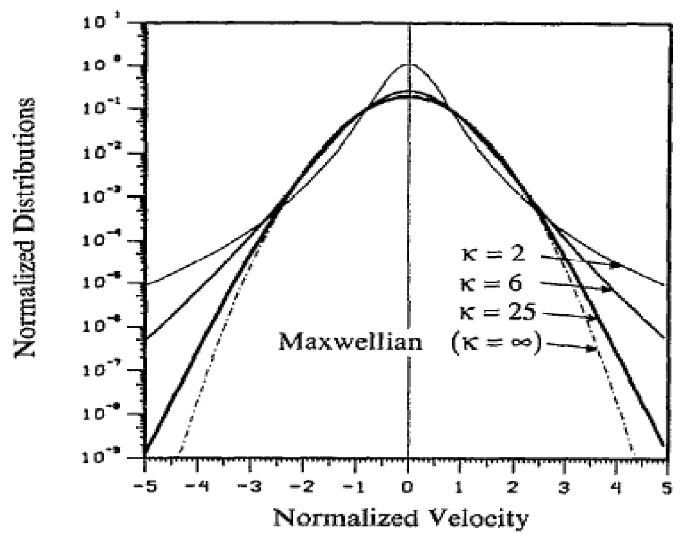

FIG. 1: Comparison of generalized Lorentzian distributions for the spectral index $\kappa = 2$, 6 and 25, with the corresponding Maxwellian distribution ($\kappa = \infty$) [7].

The scope of this study is to investigate the effect of superthermality on electron-acoustic (EA) waves, focusing on electrostatic (ES) solitary waves (ESW) (§3) and EA modulated wavepackets (§4); investigating their modulational (in)stability profile and envelope soliton modes.

2. Fluid model for electron-acoustic (EA) waves

We consider a plasma containing inertial ("cool") electrons, $\kappa-$distributed ("hot") electrons and background stationary ions. The fluid equations for the cold electrons ($\gamma = 3$) read

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n}\frac{\partial P}{\partial x}, \quad (2)$$

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + 3P\frac{\partial u}{\partial x} = 0, \quad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2} = -(\beta+1) + n + \beta\left(1 - \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa+1/2} \quad (4)$$

Normalization factors:

$$n = n_c/n_{c0}\,, \quad u = u_c/v_0\,, \quad \phi = \Phi/\Phi_0\,, \quad t = t\,\omega_{pc}$$

$$x = x/\lambda_0\,, \quad P = P/n_{c0}k_B T_c\,, \quad v_0 \equiv (k_B T_h/m_e)^{1/2}$$

$$\lambda_0 = \left(k_B T_h/4\pi n_{c0} e^2\right)^{1/2}, \quad \omega_{pc}^{-1} = (4\pi n_{c0} e^2/m_e)^{-1/2}.$$

We define the density ratio and the temperature ratio as

$$\beta = n_{h,0}/n_{c,0} \qquad \sigma = T_c/T_h\,. \quad (5)$$

3. Electrostatic solitary waves

Considering Eqs. (1)–(4) in a stationary frame travelling at a constant velocity M ($\xi = x - Mt$), we obtain:

$$u = M\left(1 - \frac{1}{n}\right), u = M - \left(M^2 + 2\phi - 3n^2\sigma + 3\sigma\right)^{1/2}, P = n^3.$$

Poisson's Eq. (4) thus leads to a pseudo-energy balance equation:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \Psi(\phi) = 0, \quad (6)$$

where the "Sagdeev" pseudopotential function $\Psi(\phi)$ reads

$$\Psi(\phi) = (1+\beta)\phi + \beta\left[1 - \left(1 + \frac{\phi}{-\kappa + \frac{3}{2}}\right)^{-\kappa+3/2}\right] + \frac{1}{6\sqrt{3\sigma}}\left[\left(M + \sqrt{3\sigma}\right)^3 - \left(M - \sqrt{3\sigma}\right)^3 - \left(2\phi + \left[M + \sqrt{3\sigma}\right]^2\right)^{3/2} + \left(2\phi + \left[M - \sqrt{3\sigma}\right]^2\right)^{3/2}\right] \quad (7)$$

3.1 Soliton Existence

In order for solitons to exist, we need to impose [8, 9]: $\Psi'(\phi) \equiv d\Psi/d\phi = 0$ and $\Psi''(\phi) \equiv d^2\Psi/d\phi^2 < 0$ at $\phi = 0$, leading to

$$F_1(M) = -\Psi''(\phi)|_{\phi=0} = \frac{\beta(\kappa - \frac{1}{2})}{\kappa - \frac{3}{2}} - \frac{1}{M^2 - 3\sigma} > 0. \quad (8)$$

Eq. (8) provides the minimum value for the Mach number:

$$M_1 = \left[\frac{\kappa - \frac{3}{2}}{\beta(\kappa - \frac{1}{2})} + 3\sigma\right]^{1/2}. \quad (9)$$

An upper limit of M can be obtained by imposing $F_2(M) = \Psi(\phi)|_{\phi=\phi_{\max}} > 0$ (where $\phi_{\max}$ is the root of $V(\phi)$):

$$F_2(M) = -\frac{1}{2}(1+\beta)\left(M - \sqrt{3\sigma}\right)^2 - \frac{4}{3}M^{3/2}(3\sigma)^{1/4} + \beta\left(1 - \left[1 + \frac{\left(M - \sqrt{3\sigma}\right)^2}{2\kappa - 3}\right]^{-\kappa+3/2}\right) + M^2 + \sigma > 0 \quad (10)$$

3.2 Parametric investigation

3.2.1 Critical Mach number M values:

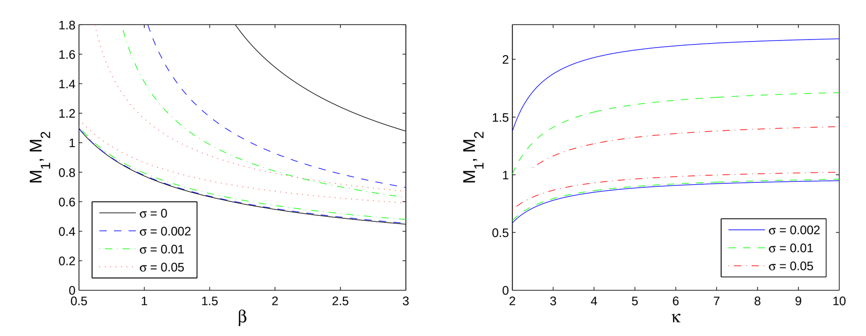

FIG. 2: Soliton existence region ($M_1 < M < M_2$) for different temperature ratio σ values, versus β for $\kappa = 3$ (left panel); also, versus κ for $\beta = 1$ (right panel).

3.2.2 Thermal effect (via σ):

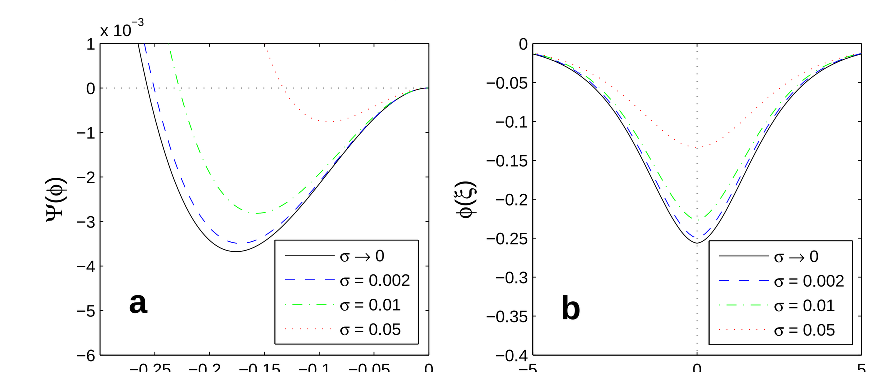

FIG. 3: (left) The pseudopotential $\Psi(\phi)$ (left) and the associated electric potential $\phi(\xi)$ (right) are depicted, for different values of the temperature ratio σ. Here, $\beta = 1$, $\kappa = 3$ and $M = 1$.

3.2.3 Superthermality and hot-cool electron effects:

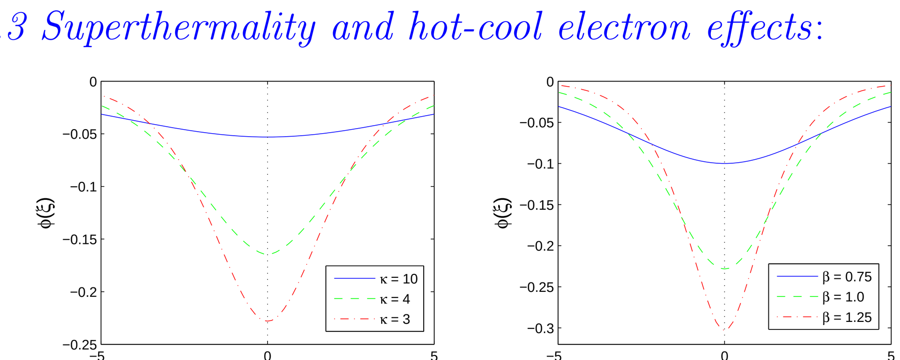

FIG. 4: Electric potential ϕ vs. ξ for different: κ (left; here $\sigma = 0.01$, $\beta = 1$, and $M = 1$); and density ratio β (right; here, $\sigma = 0.01$, $\kappa = 3$ and $M = 1$).

4. Multiscale theory for envelope modes

We anticipate modeling observed modulated envelope multi-harmonic modes as envelope solitons of the bright/dark type:

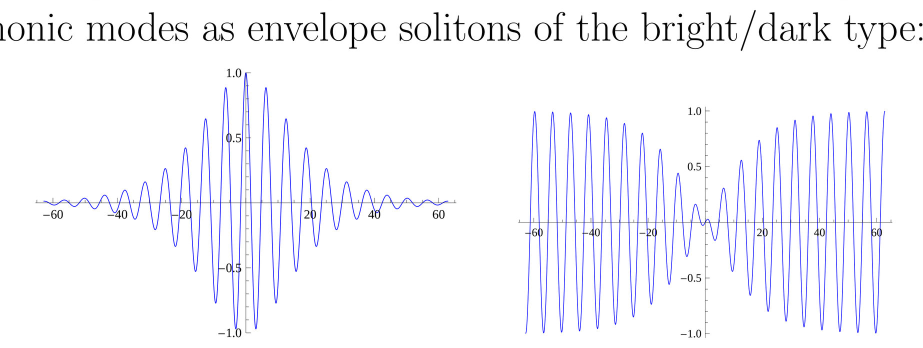

FIG. 5: Bright-type (left) and dark-type (right) envelope soliton(s) (heuristic plots).

We consider small ($\epsilon \ll 1$) deviations of all state variables, say $S = (n, u, P, \phi)$, from the equilibrium state as

$$S = S^{(0)} + \Sigma_{n=1}^{\infty}\epsilon^n \sum_{l=-n}^{n} S^{(nl)} e^{il(kx-\omega t)}.$$

In this part, we ignore the pressure term (*cold electron model*), for simplicity. We set $\alpha = n_{c,0}/n_{h,0}$ ($= \beta^{-1}$ See §3).

The 1st order ($\sim \epsilon^1$) expressions provide the EAW *dispersion relation*, along with the amplitudes of the first harmonics ($n = l = 1$):

$$\omega^2 = \frac{k^2\alpha}{k^2 + c_1}\,, \qquad n_1^{(1)} = \frac{k^2 + c_1}{\alpha}\phi_1^{(1)}\,, \qquad u_1^{(1)} = -\frac{k}{\omega}\phi_1^{(1)}\,.$$

To 2nd order ($\sim \epsilon^2$), the solution obtained has the form

$$\phi \simeq \epsilon\phi_1^{(1)} e^{i(kx-\omega t)} + \epsilon^2\left[\phi_0^{(2)} + \phi_2^{(2)} e^{2i(kx-\omega t)}\right] + \mathcal{O}(\epsilon^3)\,,$$

with a similar set of expressions for the electron density n and fluid velocity u (a complex conjugate is understood in all harmonics).

4.1 Nonlinear Schrödinger equation (NLSE) for $\phi_1^{(1)}$:

$$i\frac{\partial \psi}{\partial \tau} + P\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2\,\psi = 0\,. \quad (11)$$

where

- $\psi \equiv \phi_1^{(1)}(\zeta, \tau)$.
- $\zeta = \epsilon(x - v_g t), \quad \tau = \epsilon^2 t, \quad v_g = \frac{d\omega}{dk} = \frac{\omega^3 c_1}{k^3\alpha}$.
- Dispersion coefficient: $P = \frac{1}{2}\omega''(k) = -\frac{3}{2}\frac{\omega^5 c_1}{k^4\alpha^2}$.
- Nonlinearity coefficient: $Q \to$ long expression omitted here.

4.2 Modulational instability (MI) of ES wavepacket

- Plane wave solution of (11): $\psi = \psi_0 \exp(iQ|\psi_0|^2\tau)$.
- Linear perturbation analysis: $\tilde{\psi} = \tilde{\psi}_0 + \epsilon\,\tilde{\psi}_{1,0}\cos(\tilde{k}\zeta - \tilde{\omega}\tau)$.
- Dispersion relation *(for an amplitude perturbation* $\{\tilde{k}, \tilde{\omega}\}$*)*:

$$\tilde{\omega}^2 = P\,\tilde{k}^2\,(P\tilde{k}^2 - 2Q|\tilde{\psi}_{1,0}|^2)\,. \quad (12)$$

- The sign of the product PQ determines the stability profile.
  - If $PQ < 0$: stability (cf. (12)), *dark* envelope solitons exist,
  - If $PQ > 0$: *modulational instability* (MI), *bright* solitons.
- Modulational instability threshold: $\tilde{k}_{cr} = (\frac{2Q}{P})^{1/2}|\tilde{\psi}_{1,0}|$.
- $\kappa-$dependent MI growth rate.

4.3 Parametric analysis

4.3.1 The critical carrier wavenumber k_{cr}:

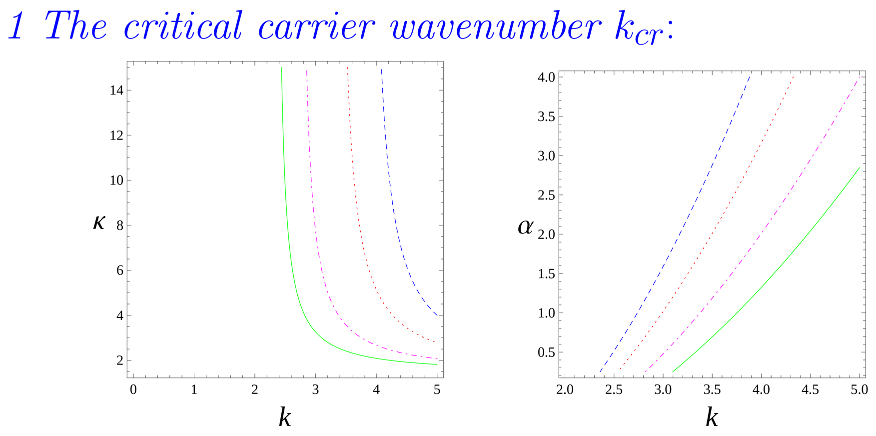

FIG 8: $PQ = 0$ (or $k = k_{cr}$) contours vs carrier wavenumber k and superthermality parameter κ (MI occurs in the right region, for $k > k_{cr}$). Left panel: α=0.25 (green); 1 (magenta); 2.5 (red); and 4 (blue). Right panel: κ=3 (green); 4 (magenta); 8 (red); and 100 (blue).

4.3.2 Superthermality and hot electron concentration effects:

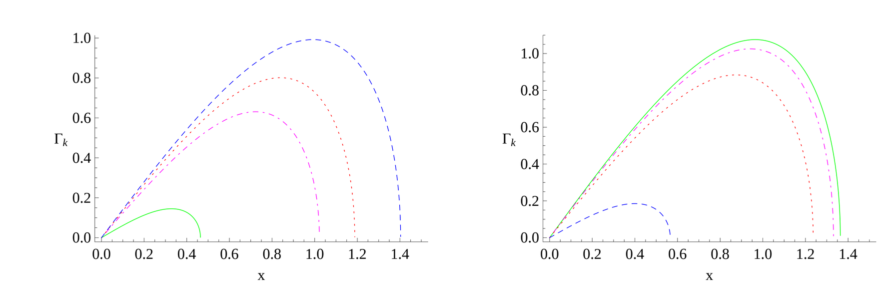

FIG. 6: The instability growth rate Γ_κ (scaled by the Maxwellian value Γ_∞) is depicted versus the (scaled) perturbation wavenumber. Left panel: $\kappa = 3.5$ (green); 5 (magenta); 7 (red); and 100 (blue) for $\alpha = 0.5$, $k = 3.2$. Right panel: green for $\alpha = 0.5$; magenta for 1; red for 2; blue for 4 for $k = 4.5$, $\kappa = 7$.

4.3.3 Bright versus dark envelope soliton:

We depict the ratio P/Q versus k for $\alpha = 0.5$ (left panel), and versus α for $k = 3$ (right panel).

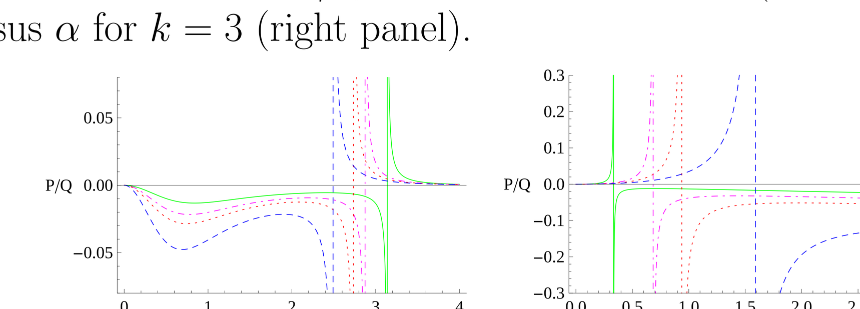

FIG. 7: P/Q vs. k for $\alpha = 0.$ (left panel), and vs. α for $k = 3$ (right panel). Modulational instability occurs for $P/Q > 0$; see above. Curves: green for $\kappa = 3.5$, magenta for $\kappa = 5$, red for $\kappa = 7$, and blue for $\kappa = 100$ (Maxwellian!).

5. Conclusions

- Increase in cold electron temperature leads to a lower excitations and subsonic region (for fixed β) for solitary wave.
- The MI wavenumber *threshold* k_{cr} *decreases* (leading to a wider stability window and bright solitons) in the presence of more superthermal electrons (via β) for fixed κ.
- An increase in the hot electrons number density leads to a higher amplitude and more unstable solitary structures (see Fig. 4b).
- Higher superthermality provides a wider stable region.
- A higher concentration of superthermal electrons gives rise to bright solitons and enables modulational instability (see Fig. 6b).

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References

[1] Marco Borghesi, Queen's University Belfast, private communication.

[2] N. Meyer-Vernet, pp. 269, in Solar Activity and its Magnetic Origin (V. Bothmer & A.A. Hady, Eds.), Proc. IAU Symposium No. 233 (2006).

[3] S. Sultana, I. Kourakis, N. S. Saini and M. A. Hellberg, *Phys. Plasmas*, in press (2010).

[4] D. Summers and Richard M. Thorne, *Phys. Fluids* **B 3**(8), 1835 (1991).

[5] V.M. Vasyliunas, *J. Geophys. Res.* **73**, 2839 (1968).

[6] R.L. Mace and M.A. Hellberg, *Phys. Plasmas* **2**, 2098 (1995).

[7] D. Summers and R.M. Thorne, *Phys. Fluids* **B3**, 1835 (1991).

[8] F. Verheest, T. Cattaert, G.S. Lakhina, and S.V. Singh, *J. Plasma Phys.* **70**, 237 (2004).

[9] F. Verheest, M.A. Hellberg, and G.S. Lakhina, *Astrophys. Space Sci. Trans.* **3**, 15 (2007).