Ion-Acoustic Solitary Waves in a Two-Electron-Temperature Plasma

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Introduction

- In e-i plasma, supersonic positive (compressive) solitary waves exist.
- Soliton maintains its shape, while it travels at constant speed.
- e_{cold}-e_{hot}-i plasma (2e-i):
    - containing two populations of electrons with different temperatures ($T_c < T_h$): described 2 different Maxwellians.
- 2e-i plasmas were:
    - observed experimentally.
      [Olesen & Found, *J. App. Phys.* **20**, 416 (1946)]
      [Jones & et al, *Phys. Rev. Let.* **35**, 1349 (1975)]
    - studied theoretically.
      [Nishihara & Tajiri, *J. Phys. Soc. Jap.* **50**, 4947 (1981)]
    - also observed by satellites in the Earth's aurora.
      [Berthomier, Pottelette, & Malingre, *J. Geo. Res.* **103**, 4261 (1998)]

Model Equations

Fluid Model for Ion-acoustic Waves

- Continuity Equation:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \tag{1}$$

- Momentum Equation:

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} \tag{2}$$

- Poisson Equation:

$$\varepsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -e\left(n_i - n_{h,0}\exp\left(\frac{e\phi}{k_B T_h}\right) - n_{c,0}\exp\left(\frac{e\phi}{k_B T_c}\right)\right) \tag{3}$$

- Ions do not include thermal effects ($T_i \approx 0$).
- quasi-neutral condition at equilibrium $n_{i,0} = n_{h,0} + n_{c,0}$.
- u_i ions fluid velocity, n_i ions density, m_i ion mass, ϕ electric potential, $n_{h,0}$, $n_{c,0}$ hot & cold electrons density, T_h, T_c hot & cold electrons temperature.

Model Equations

- Scale-reduction:

$$\frac{n_i}{n_{i,0}} \to n, \quad \frac{\phi}{k_B T_h / e} \to \phi, \quad \frac{u_i}{c_{i,s}} \to u \tag{4}$$

$$t\omega_{pi} \to t, \quad \frac{x}{\lambda_D} \to x \tag{5}$$

  where

$$c_{i,s}^2 = k_B T_h / m_i, \quad \omega_{pi}^2 = n_{i,0} e^2 / \varepsilon_0 m_i, \quad \lambda_D^2 = \varepsilon_0 k_B T_h / n_{i,0} e^2 \tag{6}$$

- Density ratio:

$$\rho = \frac{n_{c,0}}{n_{h,0}} \tag{7}$$

- Temperature ratio:

$$\theta = \frac{T_c}{T_h} \tag{8}$$

- For $\rho \to 0$ (No cold e's.), we recover e-i plasma [PHY9013 lectures].

Model Equations

Dimensionless Model

- Continuity Equation:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \tag{9}$$

- Momentum Equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{10}$$

- Poisson Equation:

$$\frac{\partial^2 \phi}{\partial x^2} = -\left(n - \frac{1}{(1+\rho)}\exp(\phi) - \frac{\rho}{(1+\rho)}\exp\left(\frac{\phi}{\theta}\right)\right) \tag{11}$$

- Anticipated localization of Electrostatic Energy

$$n = 1, \quad u = 0, \quad \phi = 0, \quad E = 0, \quad \text{(equilibrium)}$$

Nonlinear Pseudopotential Technique

- Traveling coordinate:

$$X = x - Vt \tag{12}$$

- Replacing the derivatives:

$$\frac{\partial}{\partial x} = \frac{d}{dX}, \quad \frac{\partial}{\partial x} = -V\frac{d}{dX} \tag{13}$$

- Integrating, and assuming at infinity ($n = 1$, $u = 0$, and $\phi = 0$):

$$u = V\left(1 - \frac{1}{n}\right), \quad u = V - \left(V^2 - 2\phi\right)^{1/2} \tag{14}$$

- We obtain the ion density:

$$n = \left(1 - \frac{2\phi}{V^2}\right)^{-1/2} \tag{15}$$

Nonlinear Pseudopotential Technique

Energy Balance Equation:

- We obtain

$$\frac{1}{2}\left(\frac{d\phi}{dX}\right)^2 + \Psi(\phi, V) = 0 \tag{16}$$

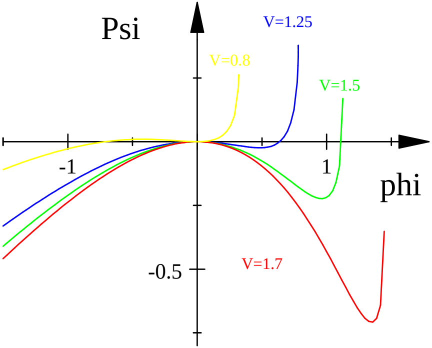
- Pseudopotential:

$$\begin{aligned}\Psi(\phi, V) &= V^2\left(1-\left(1-\frac{2\phi}{V^2}\right)^{1/2}\right)+\frac{1+\rho\theta}{1+\rho}\\ &\quad -\frac{1}{1+\rho}\left(\exp(\phi)+\rho\theta\exp\left(\frac{\phi}{\theta}\right)\right),\end{aligned} \tag{17}$$

- Parameter space:
  Velocity (Mach number) V, Density ratio ρ, Temperature ratio θ.

Nonlinear Pseudopotential Technique

- The known e-i case ($\rho = 0$)



- Solitons exist in the range: $1 < V < 1.58$.

Soliton Existence

- Two Thresholds V_1, V_2: so $V_1 < V < V_2$
- $f_1(V) = \Psi''(\phi, V)|_{\phi=0} \leqslant 0$: (for soliton existence)

$$V \geqslant V_1(\rho, \theta) = \left(\frac{1+\rho}{1+\rho/\theta}\right)^{1/2} \tag{18}$$

- $f_2(V) = \Psi(\phi, V)|_{\phi=V^2/2} \geqslant 0$: (for reality of the solution)

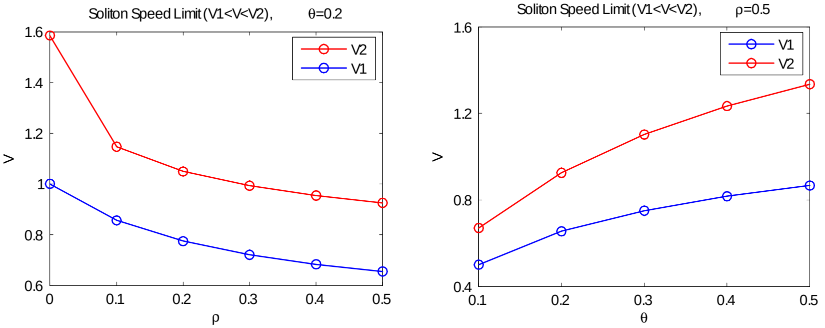
$$V_2^2 + \frac{1+\theta\rho}{1+\rho} - \frac{1}{1+\rho}\left(\exp\left(\frac{V_2^2}{2}\right) + \theta\rho\exp\left(\frac{V_2^2}{2\theta}\right)\right) \geqslant 0 \tag{19}$$

(to be solved numerically e.g. Newton method)

- For $\rho \to 0$ (e-i plasma), we recover : $V_1 = 1$ and $V_2 = 1.585$.

Soliton Existence

Mach Number Range



- The range of V values decreases with increasing $\rho = n_c / n_h$.
- The range of V values increases with increasing $\theta = T_c / T_h$.
- In 2e-i plasma subsonic solitons $(V < 1)$ exist, while subsonic solitons are impossible in a e-i plasma $(1 < V < 1.58)$.

Parametric Investigation

Small Amplitude Limit for Soliton Pulse

- expanding the pseudopotential as

$$\Psi(\phi, V) \approx \underbrace{\frac{1}{2}\left(\frac{1}{V^2} - \frac{1+\rho/\theta}{1+\rho}\right)}_{a}\phi^2 + \underbrace{\frac{1}{2}\left(\frac{1}{V^4} - \frac{1+\rho/\theta^2}{3(1+\rho)}\right)}_{b}\phi^3$$

  (approximation)
- solving analytically

$$\tfrac{1}{2}\phi'(X)^2 + a\phi^2 + b\phi^3 = 0 \tag{20}$$

- This shows

$$\phi = \phi_{\mathrm{max}}\mathrm{sech}^2\left(-\frac{X}{W}\right) \tag{21}$$

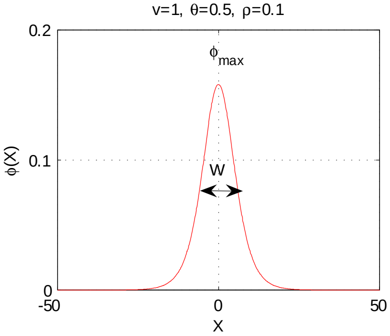
  where

$$\phi_{\mathrm{max}} = -\frac{a}{b}, \quad W = \sqrt{\frac{2}{a}} \tag{22}$$

Parametric Investigation

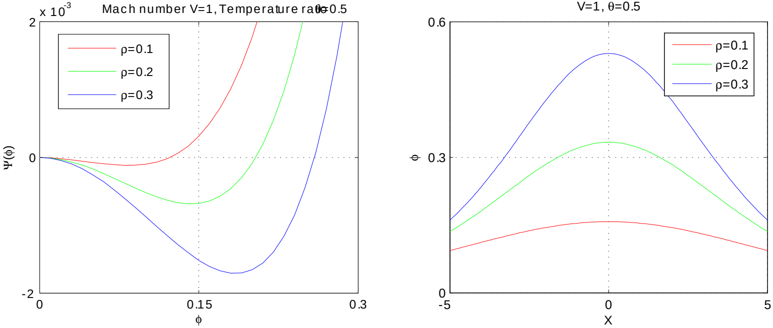
Small Amplitude Limit for Soliton Pulse

$$\phi = \phi_{\max}\operatorname{sech}^2\left(-\frac{X}{W}\right)$$



Parametric Investigation

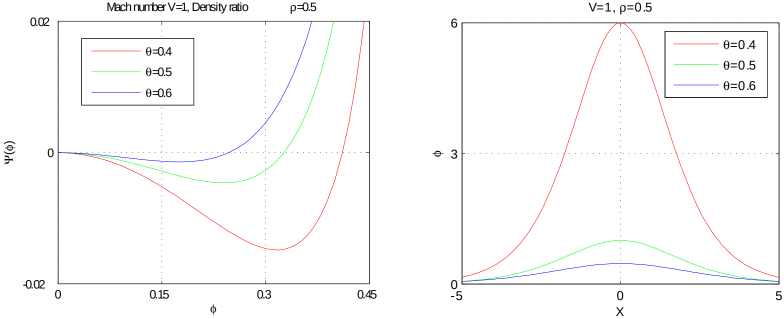
Effect of Cold Electron Density (varying ρ for $\theta = 0.5$)



- For fixed θ, the maximum amplitude of potential ϕ increases with increasing ρ (more cold electrons).

Parametric Investigation

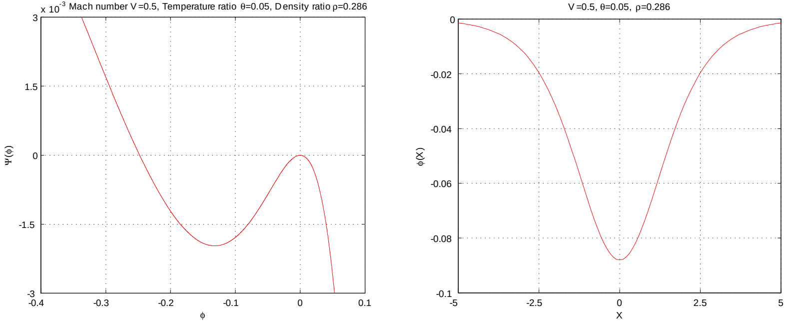
Effect of Cold Electron Temperature (varying θ for $\rho = 0.5$)



- For fixed ρ, the maximum amplitude of potential ϕ increases with decreasing θ (colder e$_{\text{cold}}$).

Parametric Investigation

Novel possibility: Rarefactive IA soliton!



- $n = \left(1 - \frac{2\phi}{V^2}\right)^{-1/2}.$
- $\begin{cases} n > 1 \quad \rightarrow \quad 0 < \phi < \phi_{\max} : & \text{Compressive Soliton} \\ n < 1 \quad \rightarrow \quad \phi < 0 : & \text{Rarefactive Soliton} \end{cases}$
- Agreement with [Nishihara & Tajiri, *J. Phys. Soc. Jap.* **50**, 4047 (1981)]

Conclusion

- e-i plasma:
  - Mach number range: $V \in [1, 1.58]$.
  - $1 < V$ → only supersonic positive solitons exist.
  - compressive positive soliton $0 < \phi < \phi_{\max} = V^2/2$.
- e_c-e_h-i plasma:
  - Mach number range: $V \in [V_1, V_2]$.
  - $V_1 = \left(\frac{1+\rho}{1+\rho/\theta}\right)^{1/2}$ → subsonic ($V < 1$) is also possible, confirmed by experiments: [Olesen & Found, *J. App. Phys.* **20**, 416 (1946)] observations: [Berthomier, et al, *J. Geo. Res.* **103**, 4261 (1998)]
  - for some special cases show rarefactive solitons: $n < 1$, $\phi < 0$. [Nishihara & Tajiri, *J. Phys. Soc. Jap.* **50**, 4947 (1981)]
- In e_c-e_h-i plasma, increasing $\rho = n_c/n_h$ or decreasing $\theta = T_c/T_h$
  - decreases range of Mach number V.
  - increases the maximum amplitude of potential ϕ.

Conclusion

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Conclusion

References

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Thank You!